“Playing the Game” of Story Problems: Situated Cognition in Algebra Problem-Solving

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Abstract

Several justifications have been presented in the literature for teaching mathematics in contexts relevant to students; first, embedding mathematics in relevant contexts may help students to apply what they learn in school to the real world. Second, using relevant contexts may provide a bridge between what students already understand and the content they are trying to learn. In the present study, we examine these justifications using algebra story problems on linear functions. In a series of 24 clinical interviews, students from a low-performing urban school were presented with algebra problems, some of which were personalized to the ways in which they described using mathematics in their everyday lives. We found that students rarely activated real world knowledge when solving all types of story problems, had consistent issues with verbal interpretation of stories, and engaged in non-coordinative reasoning where they bypassed the intermediate step of understanding the problem situation before trying to solve the problem. However, some students engaged in sophisticated situation-based reasoning, while others seemed to accept that a lack of sense-making was part of the larger system of school mathematics.

Theoretical Framework

In the ***symbol precedence view***, algebraic symbolism is presented first during instruction, and story problems are then used as a way to apply these formalisms. In the ***verbal precedence view***, verbal skills develop before symbol skills, and thus story problems are presented before symbolic equations (Nathan & Petrosino, 2003). From a symbol precedence perspective, the purpose of story problems is to solve the “transfer problem”; by giving students contextualized problems, they will be prepared to face the demands of using math in the real world. From a verbal precedence perspective, context may provide accessibility or scaffolding for students. Studies have shown that while many teachers and textbooks subscribe to symbol precedence views, students’ performance better corresponds to a verbal precedence model (Koedinger & Nathan, 2004; Nathan & Koedinger, 2000a; 2000b; Nathan, Long, & Alibali, 2002; Nathan & Petrosino, 2003). However, research on situated cognition (e.g. Greeno, 2006) and arithmetic story problems (Greer, 1997; Inoue, 2005; Kazemi, 2002; Palm, 2008; Saxe, 1988; Taylor, 2005) has called into question whether the common justifications behind either of these two models are complete given the situated nature of problem solving, and the complex, social systems that students participate in as they learn mathematics in school.

In Nathan, Kintsch, & Young’s (1992) model of algebra story problem comprehension, students must coordinate three levels of representation when solving story problems: (1) the *textbase*, a propositional representation of the information in the problem, (2) the ***situation model***, a mental representation of the relationships, actions, and events in the problem, and (3) the ***problem model***, a mental representation of formal algebraic structure of the problem, involving variables and equations. We use this model to frame our study.

Methods

Problem-solving interviews of high school Algebra I students from a diverse urban district in Texas were conducted; the school had a student population with 50% economically disadvantaged and had been named “Academically Unacceptable” in mathematics. Of the 24 students in the study, 13 (54%) were Hispanic, 8 (33%) were White, and 3 (13%) were African-American. Each student participated in a 15-minute entrance interview where they were asked how they use math in their everyday lives. A set of 5 algebra problems on linear functions was then written; two of the problems in the set were personalized according to how the student described using numbers in their everyday life, while the other problems were of varied types (see Table 1). In the first two parts of each problem, the student was asked to solve for y given a specific x-value – “result unknowns.” The student was then asked to write an algebra rule representing the story, and finally was asked to solve for x given a specific y-value, a “start unknown.” Students solved their problems in an interview lasting 30 minutes to 1 hour.

Type	Example
Normal Story Problem	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, had 80 wampum shells, and spends 6 of them every day. How many shells did Tagawininto have after 10 days?
Story with Equation	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, has a number of wampum shells given by y=80-6x, where x is the number of days that have passed. How many shells did Tagawininto have after 10 days?
Personalized Story Problem	You are playing your favorite war game on the Xbox 360. When you started playing today, there were 80 enemies left in the locust horde. You kill an average of 6 enemies every minute. How many enemies are left after 10 minutes?
Generic Story	You have 80 objects, and lose 6 every day. How many objects will you have after 10 days?
Abstract	y = 80 – 6x. If x=10, what is y?

Table 1. Types of problems given to students (shows result unknowns only)

Interviews were transcribed in the NVivo software, and blocked such that one block of the transcript corresponded to a student working one problem part or answering an interviewer question. Blocks were coded with categories relating to whether students obtained intended or unintended answers, what strategies were used and what mistakes were made, evidence that real-world knowledge was activated, issues with verbal interpretation of stories, students’ use of non-coordinative methods that included a clear bypass of forming a situation model, and the creation of symbolic equations disconnected from how the student solved other parts of the problem. Cohen’s kappas (Landis & Koch, 1977) above 0.79 were obtained (substantial/almost perfect agreement).

Listening to Students’ “Real” Algebra Stories: How Students Use Numbers in Everyday Life

Many scenarios students gave when asked how they used math in their everyday life were arithmetic in nature (e.g. “counting money”), however with probing we discovered real life situations that students may think about in terms of rate of change.

“When I'm downloading music from the computer, or when I'm putting software on the computer... when it's loading it gives a time. “

“I use math at work - I use it sometimes when someone rents a table I have to see how much they pay to rent.”

“There's stuff like, this unit has 1000 health and does 100 damage per attack. And then the other units have they might have 10,000 health and they might to 20 damage per attack. If I have them attack each other, who will win?”

“I text a lot. Like as soon as I get home I start texting, and I already have like 15 messages in like 20 minutes.”

“Let's say I was making a CD, how much music would I put on the CD until... how much time is left on the CD? So you know how much the length of each song would be.”

Problem Solving Analysis: How Different Students Interpret & Solve the Same (Normal) Scenario

Issue with Verbal Interpretation:
“It’s asking for, calculate the bill for next month, and the 43 minutes for this month wouldn’t really do any - wouldn’t really affect next month’s bill.”

Issue with Verbal Interpretation/Activation of Situational Knowledge (Disruptive):
“I really don’t understand at the beginning , due to a billing error last month, I don’t get that... I think that they’re talking about, I think credit… that they get extra time to be able to pay what they owe.”

Activation of Situational Knowledge (Productive):
“I did something wrong, I got -0.8999. Because if you talked for so many minutes and she got 38.13, she must have talked more, a whole lot more… so it can’t be negative, that means she didn’t talk anything.”

Informal Strategy:
“So there was 38.13, you’re gonna add the 7 from the credit, plus 7.87. 38.13 plus 7.87 equals to $46, it equals to 46, and now you’re going to divide those, 46 divided by 0.23, which would equal to 200.”

Problem Text

Due to a billing error last month, Amanda has received a $7.87 credit towards next month's cellular phone bill. She pays a flat $0.23 per minute with no additional monthly charge.

a) If 43 minutes are used, calculate the bill for the next month.
b) If 100 minutes are used, calculate the bill for next month.
c) Write an algebra rule that represents this situation.
d) After finding that billing error last month, this month Amanda will make sure that her bill is correct. If her bill is for $38.13, how many minutes has she talked?

Situation Model

$$ Credit – Amanda gains money
$$$$$ Bill – Amanda loses money

The more Amanda talks, the larger her bill is.

Problem Model

y = 0.23x – 7.87

Problem Model

y = 0.23x – 7.87

Solution Phase

Student Believes Problem is Multiple Choice (Non-Coordinative)
“(a) says if 43 minutes are used, calculate the bill for the next month. And I was just putting in 43 times .23. I came up with 9.89 so that doesn’t work, because I think it’s trying to find how many minutes she can be on the phone to add up to 7.87. And (b) is wrong because it’s even more minutes than 43 minutes. I’m gonna go with (d) because it says if her bill was 38.13 dollars, and it’s trying to find how many minutes she’s talking and so you just divide it by .23, and I came up with 165.7 minutes.”

Non-Coordinative Approach:
“Write an algebra rule that represents the situation. So 0.23 times x equals 7.87. No… Yeah… No - that’s not what I did to get these two answers here.”

Formal Strategy (Non-Coordinative):
“It says if 43 minutes are used, I’m going to put 43 = 7.87 - 0.23x, and now I’m going to divide everything by 43. “

Students’ Conceptions of Story Problems in Algebra Class

Students found personalized problems easiest (82% of the time)

“Well, that made me remember what I worked on, so I did what I usually do at my work.”

“In the game one it was a little bit easier to connect with what it was talking about.”

Students found other problems hardest (most often abstract and generic)

“It didn’t give much information, and it's not interesting. Like, when the problem is interesting you want to figure it out because you’re curious to find out, it seems like it’s something that you want to know, you’re not just doing because you’re asked to do a question.”

“Oooh... if x equals 3, what is y? I don't like these problems! I don't know how to do these problems.”

Most students (80%) believed they had to learn about story problems in algebra because they teach practical, real-life skills

“Well first you need to know if you have a bank account, if they're messing up your money , you’re going to be short the money, and later in life you're going to be like ‘Where's all my money?’ And you just need to know it because it's... part of life.”

“Because when you get a job, or you get older, they ask you about this right? If you get a job inside like as a teacher or anything, I'm sure they would. Maybe get a job like construction work, I don’t think they would.”

“Because they’re on the TAKS?”

Objectives

The overarching purpose was to engage in an exploratory study of both the affordances and constraints of algebra story problems. More specifically, our purpose was threefold:

1. We examine personalization of story contexts and its impact on problem solving.
2. We examine the common justifications for putting mathematics in context, such as providing accessibility.
3. We discuss students’ strategies to solve algebra story problems, their use and understanding of symbolic representations, and how these practices are imbedded in the larger system of school mathematics.

Results

Problem Type	% Correct
Normal	44%
Story w/ Equation	61%
Personalized	60%
Generic	59%
Abstract	45%

Table 2. Success rates by problem type

- Students obtained the intended answer 51% of the time, with abstract and normal problems being most difficult, and personalized, story with equation, and generic problems being easiest. (Table 2).
- We noticed that our 9th graders were consistently struggling to interpret the stories; out of the 24 students, 22 had 1-7 reading interpretation issues each during their interview.
- We coded students’ activation of real world knowledge during problem solving by looking for instances where students were generating inferences based on their everyday experiences; only 20 instances were found out of 500 blocks. These activations were disruptive or unhelpful as often as they facilitated problem solving, and did not vary dramatically across problem types.
- For start unknown problems, students used informal ***trial and error*** approaches where they plugged x-values into the equation or story to get the given y-value. Students also used ***unwind*** approaches where they began with the given y-value, and reversed the slope and intercept arithmetically; we refer to this as a “transition strategy” because students are showing some understanding of systemically going backwards in a function. Students occasionally (5 of 24 students) used the formal strategy of ***equation solving***, performing operations on both sides of a symbolic equation to isolate the x variable.
- There were clear differences across problem types in strategies used (Figure 1). More concrete problem representations like personalized and normal problems elicited more informal and transition strategies, as well as lower no-response rates.
- An interesting finding is the difference between the abstract and story with equation problems; in both problem types students have to deal with a symbolic representation in the same way, but students were much more willing to work the problem when the symbolic equation was with a story context.

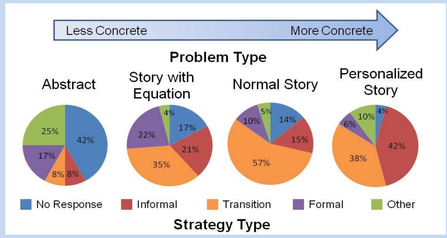

Figure 1. Students’ strategies to solve start unknowns by problem type

- We observed problem-solving approaches that were ***non-coordinative*** – students translated from the text to a problem model without developing an intermediate understanding of the situation. Some students plugged in the numbers given in the story in seemingly random orders trying to obtain an answer that “looked right.” Other students inappropriately applied a well-known schema to a problem.
- Contrary to our initial expectations, we observed that non-coordinative approaches occurred even when the problem had been personalized (15% of the time), and that this rate was similar to normal problems (12%). Story with equation, abstract, and generic problem types had higher non-coordinative rates (26-28%).

Discussion & Significance

Our findings suggest that for many students, story problems do add significant reading comprehension demands, and that these issues have a significant impact on problem solving success. Our data also demonstrates that while real world knowledge can be activated productively to provide access to story problems and error catching benefits, such spontaneous activation is rare. We also show how real world knowledge can be disruptive to problem solving. Although there have been similar findings in studies of arithmetic story problems (Inoue, 2005; Kazemi, 2002), the present study indicates significant results for algebra. Further, students’ reliance on informal, invented strategies to solve story problems provided access to the problem, and often were characterized by sophisticated, situation-based reasoning. When students did not have an interpretable story context from which to reason, they often resorted to non-coordinative approaches that were inconsistent with the situation, and relied on procedural manipulations.

We point to a need to reform algebra such that traditional story problems are no longer the primary manner in which students apply algebraic concepts to the real world. These scenarios ultimately cannot leverage students’ diverse situational understandings and mathematical sense-making activities, and do not accurately reflect the ways algebra is used in situated activity. The real story that story problems tell is not only one of verbal precedence models and efficient procedures for solutions; these problems tell us a great deal about the system in which students today learn to reason mathematically.