Context Personalization in Algebra: Supporting Connections between Relevant Stories and Symbolic Representations

Purpose

Algebra acts as a gatekeeper to higher-level mathematics (Kaput, 2000) and is essential for students' mathematical literacy and economic vitality (Moses & Cobb, 2001). However, serious problems face algebra instruction, with pressing questions being posed about how rigorous content in algebra can be effectively taught to students with diverse prior knowledge and background experiences (Allensworth, Nomi, Montgomery, & Lee, 2009; EdSource, 2009; Loveless, 2008; Loveless, Fennel, Williams, Ball, & Banfield, 2008). A recent survey conducted for the National Mathematics Advisory Panel found that when Algebra I teachers were asked to identify the single most challenging aspect of teaching algebra, the overwhelming response was "working with unmotivated students," and the second most frequent response was "making mathematics accessible and comprehensible to all of my students" (Loveless et al., 2008).

Algebra learning environments that have the potential to teach, support, and empower students have many components; however here, we focus on one – the idea of placing algebra instruction in contexts relevant to students' out-of-school experiences (e.g., Carraher, Schliemann, Brizuela, & Earnest, 2006; Chazan, 1999; Koedinger, 2001; Moses & Cobb, 2001; Noble, Nemirovsky, Wright, & Tierney, 2001). Such approaches are supported by research in mathematics education and learning sciences (e.g., Chazan, 1999; Koedinger, 2001), standards in education (NCTM, 2000; 2009; Common Core State Standards Initiative, 2010), the discourse of educational philosophy (Dewey, 1916), and the practice-based beliefs of teachers (Fives & Manning, 2005). However, few experimental studies have been conducted that isolate the effect of relevant contexts on student problem-solving, and that explore the mechanisms by which such relevance can support learning in challenging, abstract domains like algebra.

Theoretical Framework

One approach for placing instruction in contexts relevant to students' lives that is popular in computer-aided instruction is *context personalization*. In such interventions, instruction is adapted to individual students' lives and experiences, often using technology (Cordova & Lepper, 1996; Anand & Ross, 1987; Davis-Dorsey, Ross, & Morrison, 1991; Heilman et al., 2010; Reber et al., 2009).

Personalization is often hypothesized to support learning by increasing *individual* or *situational interest* (Ainley, Hidi, & Berndorff, 2002), which in turn mediate focus of attention (Durik & Harackiewicz, 2007; Hidi, 1995; McDaniel et al., 2000; Renninger & Wozniak, 1985). However, the mechanisms by which such attentional support may impact the learning of mathematics are unclear from previous research, as are the conditions under which personalization can be successful. Reading research has proposed that interest can elicit deep situational understanding (Schiefele, 1999; McDaniel et al., 2000) by supporting students' formation of *situation models* of text (Kintsch, 1986). Using case studies of three students solving personalized math problems, Renninger et al. (2002) suggest that interest may focus students' attention on story scenarios instead of keywords (which may promote *direct translation* approaches – Hegarty Mayer, & Monk, 1995), allowing learners to make connections between the story context and the mathematics content.

However, there are few personalization studies in mathematics that clearly show learning gains (Cordova & Lepper, 1996, is one exception), with some showing only immediate performance differences (Davis-Dorsey, Ross, & Morrison, 1991; Lopez & Sullivan, 1992) and others showing no differences (Bates & Weist, 2004; Caker & Simsek, 2010). In addition, the studies that have been conducted have all been within the domain of elementary school mathematics. Here, we investigate the impact of context personalization on student learning in algebra in terms of immediate performance and efficiency effects, as well as long-term learning and transfer effects. We seek to advance a theoretical understanding of how personalization is able to support learning in abstract domains like algebraic symbolization.

Methods

This paper reports on a study taking place during normal instruction as 145 ninth grade Algebra I students used the *Cognitive Tutor Algebra* software environment. Cognitive Tutor is an intelligent tutoring system where instruction is individualized through adaptive problem selection, hints, and error feedback (see Koedinger & Corbett, 2006). Participants used Cognitive Tutor as part of their algebra class two days every week. The school the participants were from was a suburban school in the northeastern United States. In terms of ethnicity, the school was majority Caucasian (96%), with 18% of students eligible for free/reduced lunch and 71% proficient in mathematics on the 11^{th} grade state standardized assessment.

Table 1

Example of variations on problem framing for story scenario in Unit 6 to correspond to different student interest categories

Original Problem	One method for estimating the cost of new home construction is based on the proposed square footage of the home. Locally, the average cost per square foot is estimated to be $46.50.
Sports	You are working at the ticket office for a college football team. Each ticket to the first home football game costs $46.50.
Music	You are helping to organize a concert where some local R&B artists will be performing. Each ticket to the concert costs $46.50.
Art	You have been working for the school yearbook, taking pictures and designing pages, and now it's time for the school to sell the yearbooks for $46.50 each.
Games	You work for a Best Buy store that is selling the newest Rock Band game for $46.50.

Participants were randomly assigned to two conditions when they entered Unit 6 of the Cognitive Tutor software, which consisted of story problems emphasizing algebraic expression-writing. The control group received the standard problems for the unit, while the experimental group received problems matched to their out-of-school interests obtained through a computer survey administered to both conditions. The personalized problems were written based on prior surveys (N=60) and interviews (N=29) with high school students. There were 27 story problems in the unit, and 4 variations on each problem were written to correspond to 9 different student interest categories (sports, music, etc.; see Table 1). For each problem, students were asked to fill in different cells of a table as they solved result and start unknowns (Figure 1). In result unknowns (questions 1-2, Figure 1), students solve for the y variable in a linear function given a specific x value - these are often solved by forward arithmetic calculation. For start unknowns (questions 3-4, Figure 1), students solve for the x variable given a y value, which may require working backwards. Students were also asked to write symbolic algebraic expressions.

An open pit copper mine is 1,550 feet deep, and the company estimates that it is getting deeper at the rate of seven feet per month. Assume the number of feet below the surface is a negative number.

1 How deep was this mine one decade ago?

2 How deep will this mine be in two years?

If you have not already done so, please fill in the expression row with an algebraic expression for the depth. Next use the expression and the Solver to answer questions 3 and 4 below.

3 In how many months will the mine be 1,564 feet deep?

4 How long ago was this mine started according to this algebraic model?

To write an expression, define a variable for the digging time and use this variable to write a rule for the depth of the mine.

Quantity Name	the digging time	the depth of the mine
Unit	month	foot
Expression	X	-1550 - 7X
Question 1	-120	-710
Question 2	24	-1718
Question 3	2	-1564
Question 4	-1550/7	0

Figure 1. Example of normal story problem scenario - upper text shows result and start unknown questions posed, while bottom table displays correct answers

Data Sources

The Cognitive Tutor software collects detailed logs of students' interactions with the system. We used hierarchical mixed effects models (Snijders & Bosker, 1999) to examine performance within Unit 6 in terms of correct answers and time measures. The level 1 unit of analysis was a student filling in one cell in Figure 1 (N=73,953), and level 2 random intercepts indicated which student was solving the problem part (N=145), which cover story was used (N=135), and which linear function (e.g., y = -1550-7x) was described in the scenario (N=27). Fixed effects included which condition the student was in (experimental or control) and which concept was being covered (e.g., solving start unknowns, result unknowns, writing expressions). We also used these models to examine student performance in the next expression-writing section, Unit 10, to see if performance differences held once the treatment was removed, indicating transfer. These models allowed for an examination of which students the treatment was most beneficial for, as well how performance was impacted on different mathematical concepts.

Results

Students in the experimental group who received personalization for Unit 6 had significantly higher performance within Unit 6, particularly on the most difficult concept in the unit, writing algebraic expressions (10% performance difference, $p<.001$). The effect of the treatment on expression-writing was significantly larger ($p<.05$) for students identified as struggling within the tutoring environment[1] (22% performance difference). Performance differences favoring the experimental group for solving result and start unknowns did not reach significance ($p=.089$). In terms of overall efficiency, students in the experimental group obtained 1.88 correct answers per minute in Unit 6, while students in the control group obtained 1.56 correct answers per minute. Students in the experimental group also spent significantly less time (p<.01) writing algebraic expressions (8.6 second reduction). However, just because personalization made problems in Unit 6 *easier* for students to solve, does not necessary mean that students *learned* more from solving the personalized problems. This distinction between performance and learning is captured by research on "desirable difficulties" (Schmidt & Bjork, 1992) which has shown that, for instance, decreasing feedback can increase learning.

Thus measures more closely related to learning were also examined. An important issue with intelligent tutoring systems is that students sometimes "game the system" (Baker, Corbett, Koedinger, & Wagner, 2004) by entering answers quickly and repeatedly, or clicking through hints until given the answer. This may reflect an orientation towards immediate performance outcomes, rather than a focus on

[1] Students were divided into four quartiles based on performance and progress measures (detailed in full manuscript). Bottom quartile students were considered struggling students.

learning the underlying concepts. Using the *Cognitive Tutor Gaming Detector* (Baker & de Carvalho, 2008), we found that within Unit 6, students in the personalization condition engaged in gaming behaviors significantly less often ($p<.05$), indicating a stronger learning orientation.

Unit 10 was the next section in Cognitive Tutor that emphasized algebraic expression-writing. Given the finding that personalization had the greatest impact for this concept, we were interested in whether differences favoring the treatment group for expression-writing would be sustained several units later with the treatment removed. We found that students who received personalization in Unit 6 were still significantly better at writing more complex symbolic expressions from standard story problems in Unit 10 (6% performance difference, $p<.05$), and this gain was significantly larger for the students who had been identified as struggling (20% performance difference, $p<.001$). Additionally, students who had received personalization still spent significantly less time writing symbolic expressions in Unit 10 (7 second reduction, $p<.05$).

Significance

The study reported here provides strong evidence that context personalization can improve performance, learning efficiency, learning orientation, and learning measures, including transfer. The positive effects were most pronounced for the most difficult concepts in the unit and for students who struggled within the tutoring environment, suggesting that interest-based interventions as adaptive assistance may be an important future direction in the development of learning technologies.

Nathan, Kintsch, and Young (1992) proposed a model of story problem solving where learners coordinate a *situation model* of the actions and relationships in the story scenario with a *problem model* of operations, symbols, and equations used to formally solve the problem. They found that when students were supported in giving algebraic equations situation-based meaning, they learned more. Here, context personalization here seemed to also promote students focusing on making meaningful connections between situations described in text related to their interests, and symbolic representations of algebraic relationships. But why did this occur?

That the effects of personalization were most pronounced for expression-writing is suggestive as to a theoretical explanation. Writing an algebraic expression from a story problem is a challenging concept for students to master (Clement, 1982; Koedinger & McLaughlin, 2010; Stacey & MacGregor, 1999) and in the context of school algebra, symbols are not always meaningful (Arcavi, 1994; Greeno & Hall, 1997). However, learning algebraic expression-writing is central to moving from concrete to abstract modes of representation - one of the major goals of algebra instruction (Kaput, 2000; NCTM, 2000). Personalization seemed to act as a kind of perceptual *grounding* (Koedinger, Alibali, & Nathan, 2008) for abstract algebraic relationships that would otherwise be challenging to represent. As a perceptual scaffold (Goldstone & Son, 2005), personalization allowed students to grasp the deeper, structural characteristics of story situations and then represent them symbolically, and retain this understanding with the support removed. This was evidenced by the transfer, performance, and efficiency effects being strongest for, or even limited to, algebraic expression-writing (even though other concepts, like solving start unknowns, were not near ceiling). This was also evidenced by the effectiveness of the treatment for supporting learning for students identified as struggling with school algebra, where a critical transition to abstract representation must be made.

Connecting instruction to students' interests and experiences has the potential to enhance learning, even in abstract domains like algebra. Embedding instruction in relevant, interest-based contexts can promote the integration of prior knowledge with formal representations by allowing learners to focus attention on this difficult task. Given the pressing challenges that face algebra instruction, designing learning environments that foster such connections could be critical to future efforts to increase access to domains where learners must navigate abstract representational systems.

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