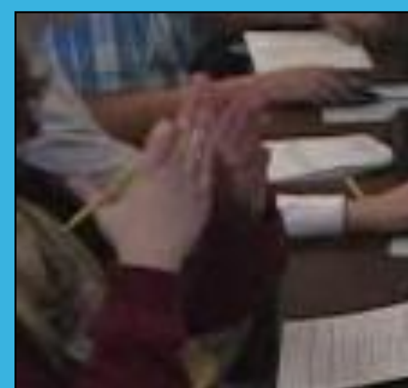
Cognition From Action

Tangibility Group - University of Wisconsin - Madison



Mary came up with the following conjecture:

"For any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side."

Provide a justification as to *why* Mary's conjecture is true or false.

RESEARCH QUESTIONS

How is action used in mathematical justification in geometry?

- Is there an *implicit* link between action and cognition that can support mathematical reasoning?

Produce

Thoughts ⟷ **Actions**

Co-constitute

BRIEF FRAMING

- The body and action important cohesion-producing mechanisms in project-based settings (DE, ME, geometry)
- Build off of previous studies on implicit link between action and cognition (Thomas & Lleras, 2007; 2009)
  - Duncker radiation problem: Participants who moved eyes in accordance with problem's solution (without realizing) more successfully solved problem.
  - Maier's two string problem: Participants who moved arms in a manner that suggested the problem's solution (without realizing) more successfully solved problem.

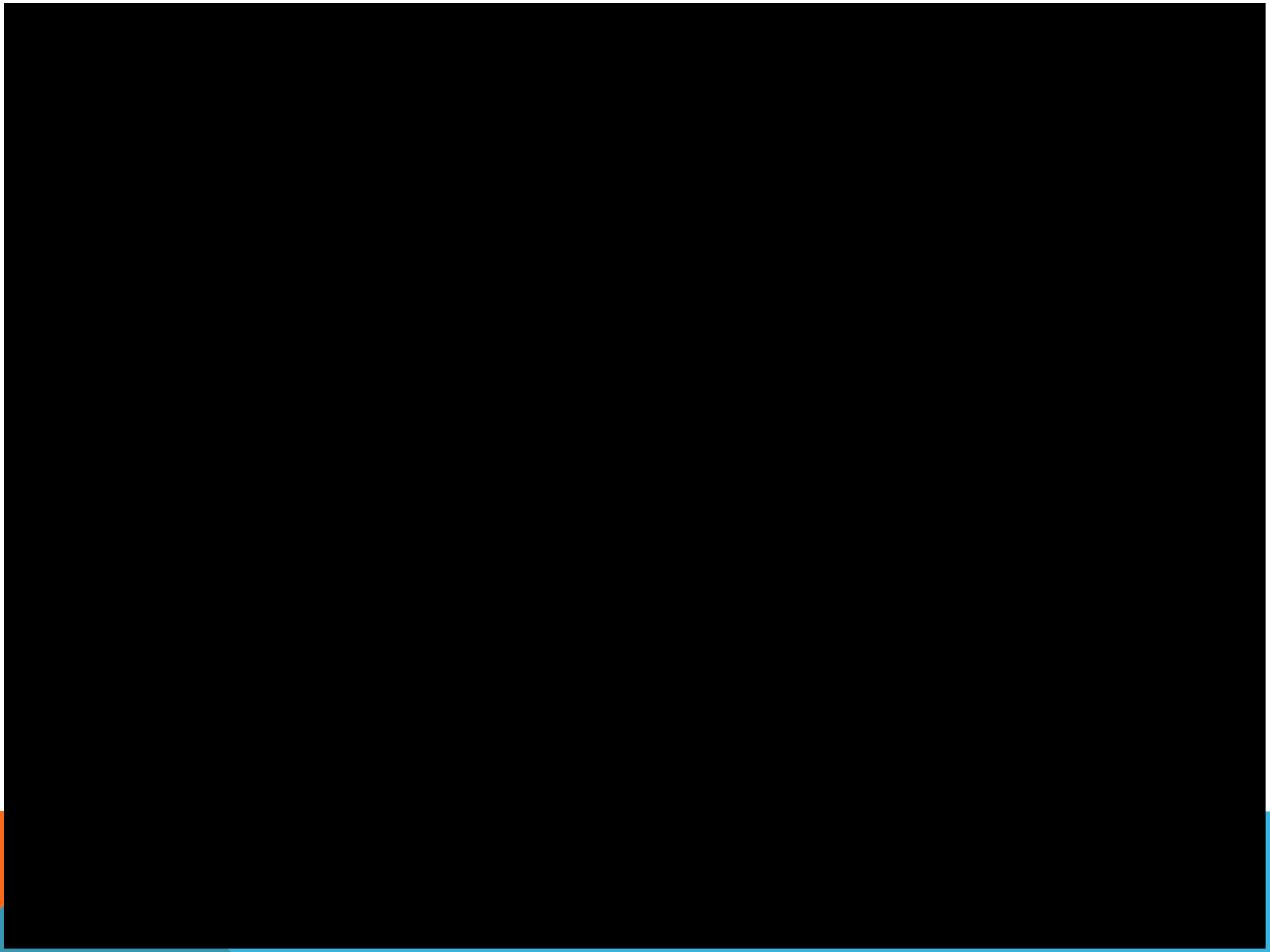
BRIEF FRAMING

- **Geometry proofs difficult to learn (Healy & Hoyles, 2000)**
  - Disconnected from typical ways of reasoning (Cooper, Walkington et al., 2011)
  - Can be taught in an amodal manner
- **Interested in an account of proof as complex multi-modal activity; emphasize the role of action**

DESIGN

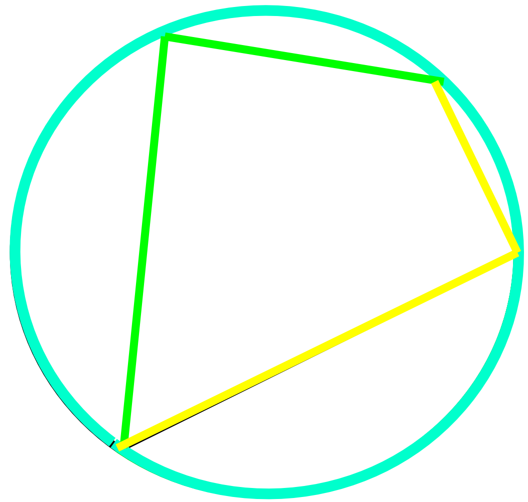
- Perform action relevant (experimental group) or irrelevant (control group) to proof of a mathematical theorem
- Provide justification for theorem and transfer theorem
- Currently, two theorems (triangle & quadrilateral), with two more being piloted (angles & rotation)

TRIANGLE TASK WITH SMART BOARD

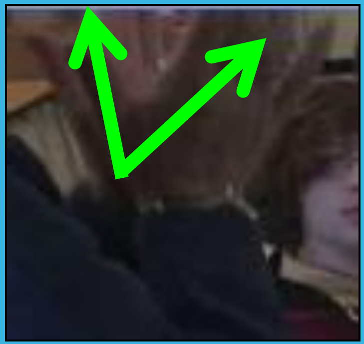


INSCRIBED QUADRILATERAL THEOREM

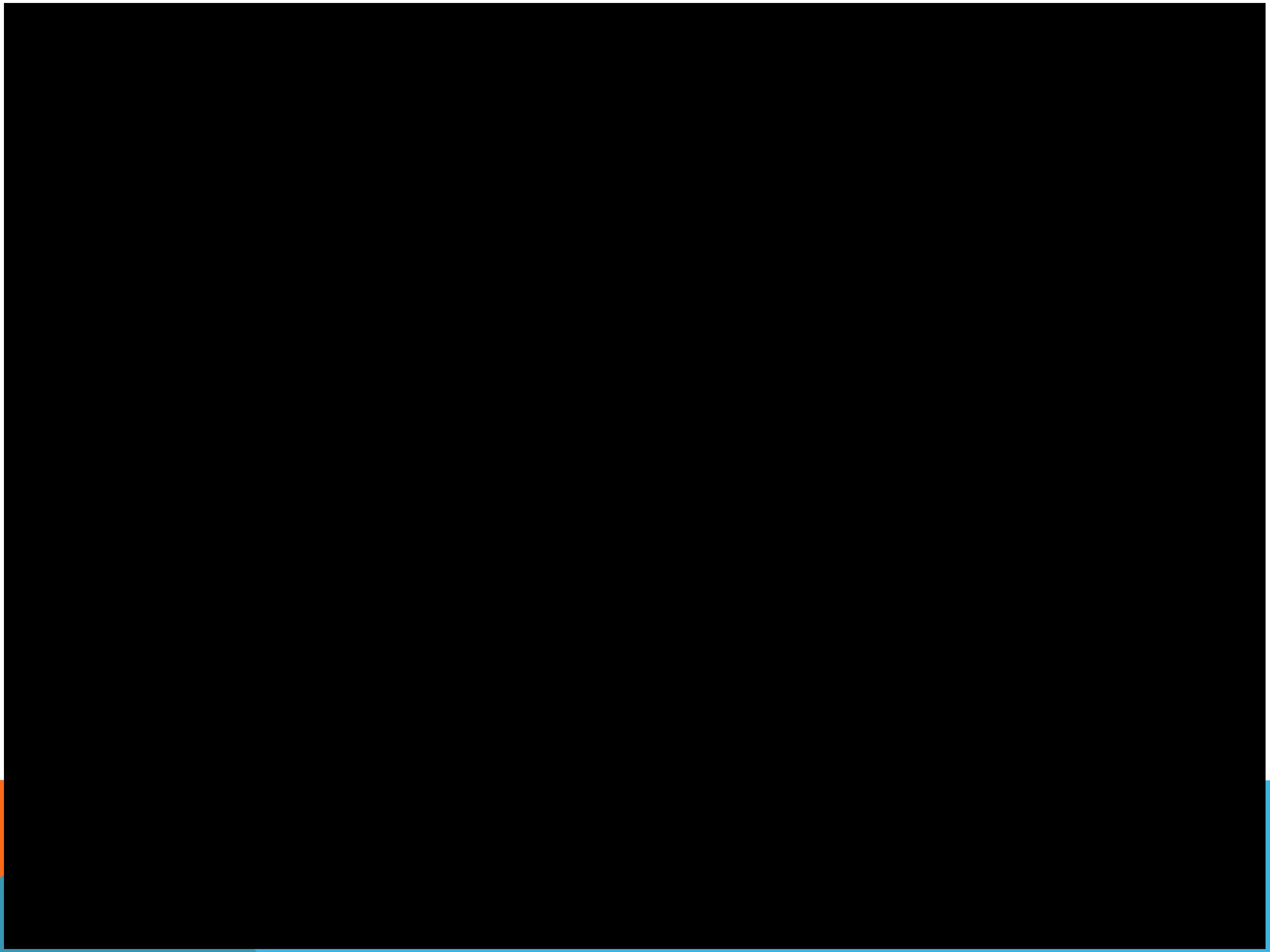
Opposite angles in an inscribed quadrilateral are always supplementary (i.e., add up to 180°).

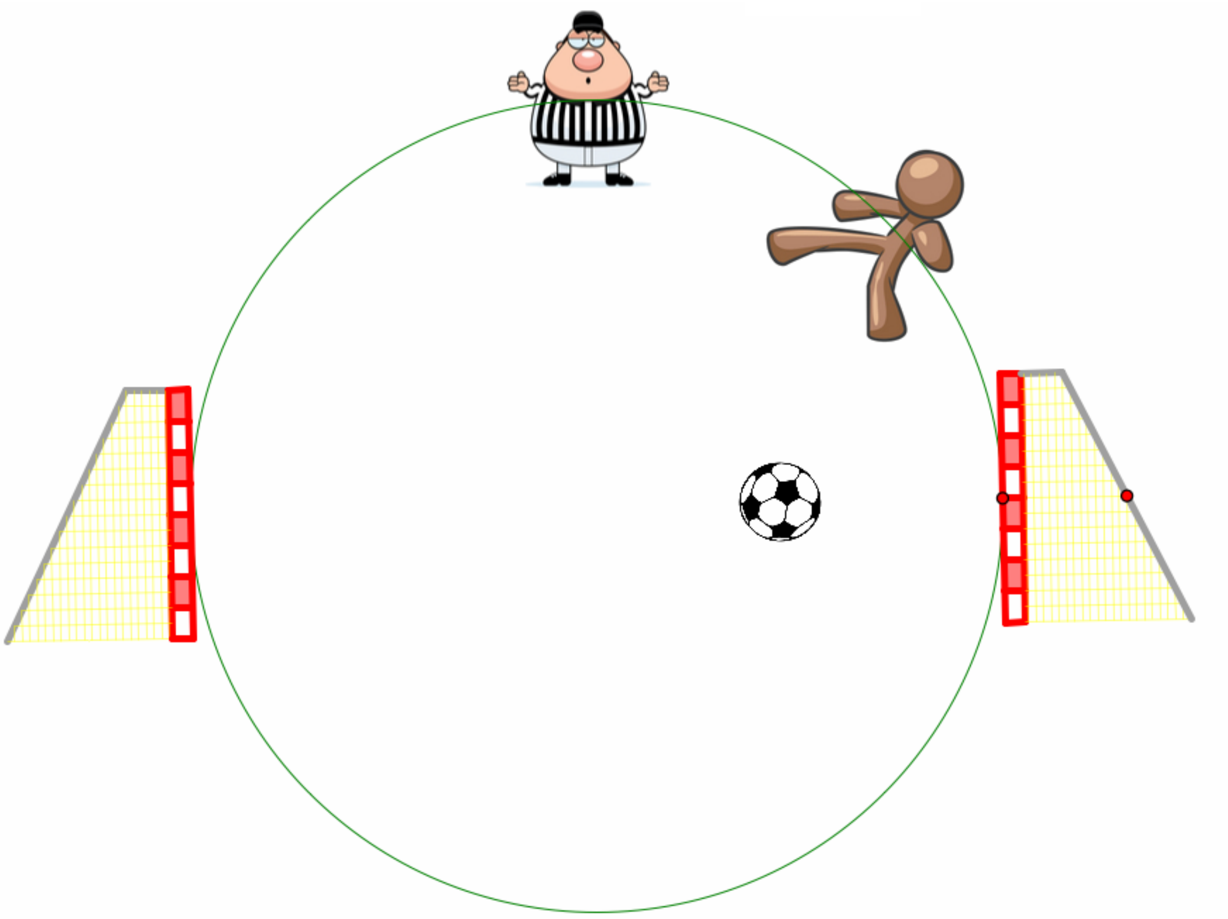


Inscribed angles and intercepted arcs have 1:2 relationship



INSCRIBED QUADRILATERAL TASK WITH SMART BOARD





Plans to more strongly accentuate contextualized, game-based nature of the interventions.

OBSERVATIONS FROM PILOT WORK

Body-based and mentally-simulated action are important tools when constructing inductive mathematical justifications.

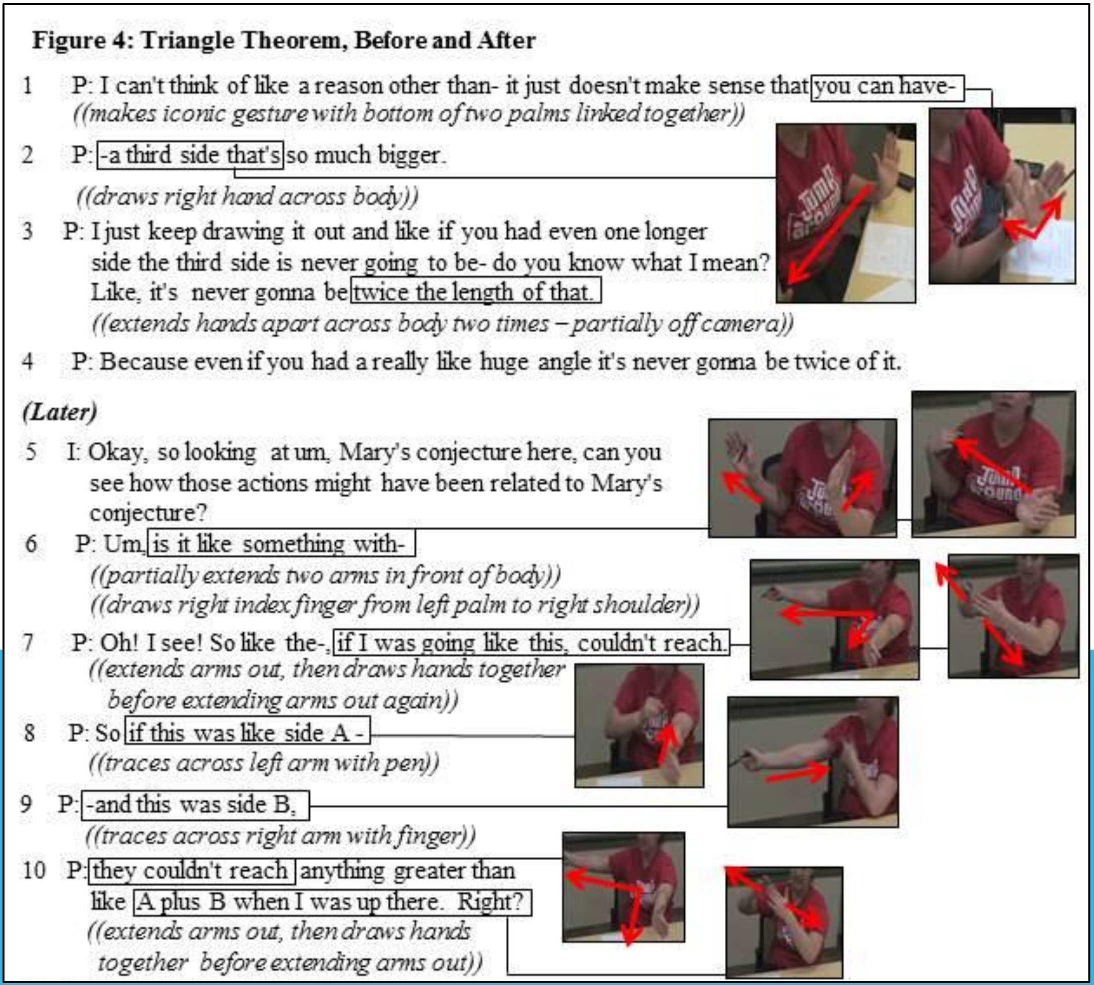


Figure 4: Triangle Theorem, Before and After

1 P: I can't think of like a reason other than- it just doesn't make sense that you can have-
((makes iconic gesture with bottom of two palms linked together))

2 P: -a third side that's so much bigger.
((draws right hand across body))

3 P: I just keep drawing it out and like if you had even one longer side the third side is never going to be- do you know what I mean? Like, it's never gonna be twice the length of that.
((extends hands apart across body two times – partially off camera))

4 P: Because even if you had a really like huge angle it's never gonna be twice of it.

(Later)

5 I: Okay, so looking at um, Mary's conjecture here, can you see how those actions might have been related to Mary's conjecture?

6 P: Um, is it like something with-
((partially extends two arms in front of body))
((draws right index finger from left palm to right shoulder))

7 P: Oh! I see! So like the-, if I was going like this, couldn't reach.
((extends arms out, then draws hands together before extending arms out again))

8 P: So if this was like side A -
((traces across left arm with pen))

9 P: -and this was side B,
((traces across right arm with finger))

10 P: they couldn't reach anything greater than like A plus B when I was up there. Right?
((extends arms out, then draws hands together before extending arms out))

OBSERVATIONS FROM PILOT WORK

Deductive reasoning also based on dynamic mental imagery.

Figure 5: Embodied Justification of Inscribed Quadrilateral Theorem

1 P: The arcs formed by the endpoints of the opposite angles for a circle, 360 degrees of arc. So the angles must always add up to 180 degrees because each angle has to account for half a circle- I don't know how to put this. Has to account for, uh, the re- the part of 360 degrees that the opposite angle doesn't.

((while writing verbatim))

2 I: Can you tell me what you mean by that last one?

3 P: Um, uh, if one angle -

((forms angle with right hand))

4 P: accounts for 100 degrees of arc -

((rolls left hand up from table to hold parallel to right hand))

5 P: the other one's gonna have to account for the remaining 3--uh--260 degrees of arc.

((curves left hand into arc, pulls towards angle formed by right hand))

(Slightly later in the justification)

6 P: The measure of the arc is twice - the angles have to add up to 180 –

((makes angle with two hands))

7 P: – so that the measure of the total arc, the total uh, circumference is 360.

((traces circle in air with pen 3 times))

OBSERVATIONS FROM PILOT WORK

Modality is not simply a medium through which a mentally-formulated sequence of arguments is transmitted. Rather, modality and justification co-constitute each other.

This suggests that we may find an implicit link between action and cognition.