Body-Based Examples When Exploring Conjectures: Embodied Resources and Mathematical Proof

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Justification and Proof

- *Are important mathematical activities (NCTM, 2000; CCSS-M, 2010)*
- *Are difficult for students (Healy & Hoyles, 2000).*

Example Usage

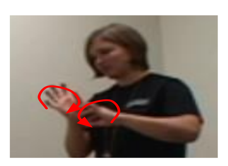
- *Examples have traditionally been dismissed as obstacles to proof production (Stylianides & Stylianides, 2009)*
- *But students can strategically use examples in order to support their justification and proof activities (Ellis et al., 2012)*

Embodied Cognition

- *Theories of **embodied cognition** posit that cognition is deeply bound up in physical action and perception (Shapiro, 2011).*
- ***Gestures** are a subset of embodied action that represent the world rather than act upon it (Goldin-Meadow & Beilock, 2010)*
- *Gestures support mathematical reasoning & communication (Alibali & Nathan, 2012)*

Generating Examples with the Body

- ***Representational gestures** represent objects or concepts using hand movements (McNeil, 1992)*


- *Body-based examples are **personalized** and **dynamic***
- ***Ground** abstract ideas in body-based form (Goldstone & Son, 2005)*

Our Line of Research

*We examine the **strategic** use of body-based examples in solving mathematical conjectures and the role of gestures in communicating mathematical ideas.*

Overarching Theme

Generating math examples with the body is beneficial for thinking meaningfully about mathematical ideas to develop general proofs.

Research Questions

1) **How do students use their bodies *spontaneously* to generate examples when constructing a mathematical proof?**
2) **How can *directing* students to generate examples with their bodies support the construction of a mathematical proof?**

Methods and Data

•90 undergraduate students enrolled in an introductory psychology course
•Two conditions: participants directed to perform gestures either relevant or irrelevant to the Triangle Conjecture
•Think-aloud protocol
•Participants videotaped by two cameras
•Video data analyzed using Transana qualitative analysis software

Triangle Conjecture

Mary came up with the following conjecture: For any triangle, the sum of the length of any two sides must be greater than the length of the remaining side. Provide a justification as to why Mary's conjecture is true or false.

Irrelevant Action

Participant taps each circle with their left palm moving from left to right, and then taps each circle with their right palm, moving from right to left.

Relevant Action

Participant places palms on pairs of circles (same colors) while keeping elbows locked and arms straight. Circles were scaled to the participant's arm span such that (s)he could not reach the far-end circles with both palms.

Coding System

•*Transformational Proof:* General, operational thought, logical inference
•*Perceptual Proof*: Invalid observations & perceptual cues
•*(Purely) Empirical Proof*: Specific examples that lack generality

RESULTS

Generating Body-Based Examples through Spontaneous Gesture

•Participants spontaneously generated examples of possible and impossible triangles through gesture
•If two sides measure 3 and 3, third side must be greater than 6 because "it wouldn't connect" – transformational proof
•Dynamic action: Triangle sides connect together and flatten out

So if a triangle has three sides,

if two sides-

let's say they are three- like, the sum of 'em is going to be six so then the third side has to be longer-

or shorter than six otherwise it wouldn't connect

to the other two sides to make

'em a triangle.

'Cause they would be like flattened out and they wouldn't fit.

Generating Body-Based Examples through Directed Gesture

•Participants used directed movements as a resource to generate relevant, body-based examples
•Formulates triangles where third side is smaller and larger than his "wingspan" when constructing a transformational proof

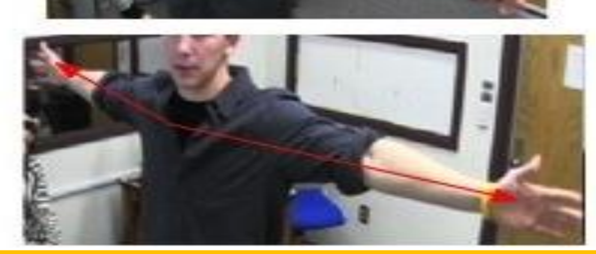
The sum of the two lengths of the two sides must be greater than the length of the remaining side. Um this is related to the thing I just did. I'd say- the sum of the lengths of any two sides must be greater than the length of the remaining side. Because if the remaining side was greater than the sum of the lengths which would be if-

if the remaining side was greater than both my arms combined that means

it would outstretch- it would be able to touch both those purple circles or out-do my wingspan

So if I think of each of my arms as a length of a side of a triangle um if- if- so my wingspan is both those arms combined uh and then the third leg has to be smaller than that because otherwise

it would be longer than my wingspan and therefore like it wouldn't be able to connect to the- the two ends of my arms.



Quantitative Trends

•**40%** of participants who performed irrelevant action generated a transformational proof
•**50%** of students who performed relevant action, but were not told of the connection, generated a transformational proof
•When informed of the connection, **70%** of students who performed relevant action generated a transformational proof
•Most of these (**62.5%**) generated a transformational proof that was explicitly grounded in the body-based examples they had been directed to generate

Conclusions & Implications

•Students can fruitfully use body-based examples for supporting mathematical justifications and proofs.
• Students may generate and manipulate mathematical objects with their gestures as way to help them "figure out" a proof.
• Such relevant actions can be taught through embodied learning activities (e.g., "being the triangle") on interactive, touch technologies
•Teachers can more deeply understand students' justifications by focusing on their gestures as well as their verbalizations.

Current Work

1. Examining Proof Processes

- Two interwoven phases: ***ascertaining*** (reason through relationships for self) and ***persuading*** (communicate argument to third party)
- Look for indicators of ascertaining and persuading in qualitative data
- Cross-referenced with gesture to identify pivotal role of gesture in formulating and communicating ideas

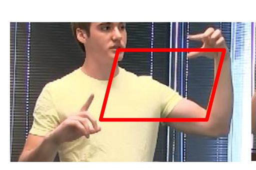
2. Exploring Dynamic Gestures

- ***Dynamic Gestures:*** Represent many possible cases through a fluid, morphing gesture
- Manipulate mathematical objects to make properties visible, like in DGS systems

The segment that joins the midpoints of two sides of any triangle, called the midsegment, is parallel to the third side.

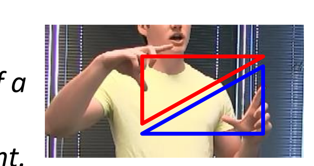
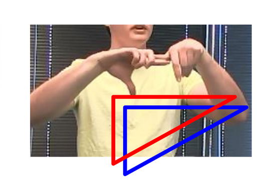

"It's just a smaller scale of the larger triangle…"
((Gestures triangle collapsing and expanding))

All four-sided figures have angles that add up to 360 degrees.



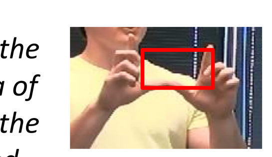

"You can't have more or less than 360 degrees. If they were any larger they wouldn't meet at the appropriate corners."
((Gestures sets of opposite angles where sides meet or do not meet))

The diagonals of a rectangle are always congruent.

"These right triangles are mirror images of each other, but they're just rotated…"
((Gestures one triangle moving on top of other))

The area of a parallelogram is the same as the area of a rectangle with the same length and height.




"The bottom stays the same, but the sides extend to keep the same height as the rectangle…"
((Gestures two vertical sides folding down))

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