

## How Students Reason Differently in Everyday and Mathematical Contexts: Typicality and Example Choice in Justification

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*Middle school students bring with them to the classroom powerful, informal resources for reasoning about mathematical ideas. However, little research has examined how these resources can interact with or support skills of mathematical justification. Here, we explore how middle school students consider inductive strategies – the use of examples in proof – when confronted with conjectures. We discuss ways in which these students might reason about mathematical objects like numbers and shapes strategically as they test examples. We argue that critical to such strategic reasoning is flexible application of mathematical and everyday ways of knowing.*

Keywords: Reasoning and Proof; Middle School Education

Students bring with them to the mathematics classroom powerful intuitive ways of reasoning based on their everyday experiences interacting with the world. An important goal in mathematics education has been to find ways to leverage these resources or “funds of knowledge” (Moll & Gonzalez, 1997) to support mathematics learning. However, this search has proven to be problematic. Students not only have trouble applying their “school math” knowledge to complex, situated real world problems (e.g., Masingila, Davidenko, & Prus-Wisniowska, 1996; Walkington, Sherman, & Petrosino, in press), but they struggle to productively use knowledge from their everyday experiences in school-based tasks (e.g., Reusser & Stebler, 1997; Walkington, Nathan, Wolfgram, Alibali, & Srisurichan, in press). A situated perspective on the nature of learning (Greeno, 2006) acknowledges that the interplay between the practices that are valued and useful in school and everyday activity is complex, and that these two sets of practices will not always overlap. However, recent work has shown promise in finding ways for students’ concrete, situated experiences to support the learning of mathematical formalisms (e.g., Moses & Cobb, 2001; Walkington & Sherman, 2012).

One area in which the interaction between everyday experience and formal mathematical knowledge has not been well-examined is mathematical justification. Here, we define justification or proving as “the process employed by an individual to remove or create doubts about the truth of an observation,” and emphasize that this process is often based on intuition, internal conviction, and necessity (Harel & Sowder, 1998, p. 243). The importance of the construction and evaluation of mathematical arguments is accentuated in both the Common Core and NCTM standards (CCSS, 2010; NCTM, 2000). But how are children’s intuitive ways of reasoning important when considering mathematical justification, which has been traditionally characterized as a formal and disembodied chain of axiomatic, deductive statements?

Recent research has revealed the *inductive* or *example-based* reasoning strategies that children use when considering mathematical conjectures (Knuth et al., 2011). For instance, when presented with the conjecture “the sum of any two even numbers is even,” a student might test

different even numbers, like 2 and 20. Some studies have suggested that this kind of inductive reasoning might allow students develop more general arguments (Knuth et al., 2011). In the study presented here, we will examine how students' everyday and mathematical knowledge interacts with their evaluation of example-based justifications in middle school mathematics. We argue that students must navigate along a learning continuum as they gain expertise with mathematical arguments, which ultimately leads to flexible and appropriate application of everyday and mathematical knowledge. Gaining an understanding of this continuum, of the ways in which students think about the nature of evidence in inductive justification, may help mathematics educators in better supporting students' learning to prove.

## **Theoretical Framework**

### **Use of example objects in justification**

Many of the problems people face in life resist formal solution. There is no deductive proof for beliefs about friends, nor a valid algorithm for picking a spouse. Instead people must employ inductive reasoning strategies. Some of the most well studied inductive strategies in the cognitive science literature are example-based (see Feeney & Heit, 2007). One way to decide if a person will be a good friend is to compare them to others. But which others? Children and adults employ a number of strategies for selecting good examples in their everyday lives, strategies that often are in line with formal principles of inductive inference.

In mathematics, students also tend to use this inductive reasoning when confronted with mathematical conjectures (Chazan, 1993; Healy & Hoyles, 2000; Knuth et al., 2011; Harel & Sowder, 1998). This approach has been identified as problematic because students may use *only* examples to justify a conjecture, without moving towards more powerful general arguments. Despite the limitations of "proof by example," examples may still play a critical role in understanding mathematical properties and constructing more general justifications. For instance, research has shown mathematicians use examples as tools for reasoning when confronted with mathematical conjectures (Alcock, 2004). In a recent survey, expert mathematicians ( $N = 133$ ) indicated they use examples to verify and understand conjectures, generalize from examples to a proof, and seek counter-examples or try to "break" the conjecture (Lockwood et al., this volume). Thus examples seem to play an important role in the development of mathematical proofs.

### **Typicality and example choice in non-mathematical domains**

In scientific domains, three principles of example selection (see Osherson et al., 1990) have been identified as useful when drawing conclusions about a class or type: *quantity* - more examples are better than fewer, *diversity* - a wide variety of examples are better than a set of very similar examples, and *typicality* - generic or 'average' examples are better than special or 'weird' examples. Thus in trying to decide whether all birds have hollow bones, one would want to check many birds, a diverse set of birds, and relatively typical birds. Here we focus on typicality - a typical example shares properties with many members of its class and has few distinctive properties. One challenge in developing accounts of example-based inference is identifying just which features are used to compute typicality. In science, people have robust intuitions about features that are 'biologically relevant'. That cats and goldfish are both kept as pets does not seem relevant in determining their biological relatedness. However, untangling everyday notions of typicality from typicality based on properties of *mathematical* objects may be more difficult.

### **Typicality and example choice in mathematics**

In previous work, we have found it useful to differentiate two types of mathematical typicality (Williams et al., 2011). The *everyday typicality* of an object is how common that object

is in everyday life – i.e., how many experiences a person has had with objects of that kind in their day-to-day activities. The *mathematical typicality* of an object is how typical the object is when its mathematical properties are considered in relation to the properties of *all* objects of that type. For example, the number “0” would be an *atypical* number in a mathematical context, because it has mathematical properties that no other integers share (e.g., additive identity). The number 322 might be a *typical* number in a mathematical context, because it does not have many mathematical properties that make it distinct from the entire set of whole numbers. Middle school mathematics is an interesting site for exploring these two types of typicality, as many of the objects that are highly *atypical* mathematically (e.g., numbers like 0 or 1) are highly *typical* in everyday life. This creates a challenge for students, as they struggle to make sense of these two very different conceptions of typicality. But do judgments of typicality really matter when considering mathematical justifications?

When the expert mathematicians ( $N = 133$ ) were asked how they choose examples when exploring conjectures, many responses referenced ideas related to the typicality of their examples. They reported choosing common examples with no special properties or generic or general examples, unusual, obscure, or “tricky” examples, examples with special properties, and examples that are boundary cases (Lockwood et al., this volume). These mathematicians seemed to have found ways to use typicality *strategically* – to allow typicality judgments to support and inform their exploration of mathematical conjectures. But what about middle school students? Do they consider typicality when exploring conjectures, and if so, what type of typicality?

In a series of interviews, we presented middle school students ( $N = 20$ ) with conjectures about properties of numbers, and students reported purposefully varying the typicality of the examples they chose when testing the conjectures. In other words, students often reported trying to test both typical and atypical numbers, or trying to test an unusual number to see if the conjecture would still hold (Cooper et al., 2011). Students’ reports of what made a number typical or atypical varied – some students were attuned to whether the number was prime, what factors the number had, or the relative size of the number. Other students identified numbers as being typical based on their everyday experiences. From this sample, it seemed that some students were reasoning strategically about the typicality of their examples. In the study presented here, we followed up on these results by implementing a large-scale survey to assess how students utilize mathematical and everyday typicality when considering examples in justification.

### **Research questions**

Our research questions were: 1) How do middle school students use typicality *strategically* when considering examples? and 2) How are students’ conceptions of mathematical typicality consistent or at odds with their everyday notions of typicality?

There are two dimensions along which middle school students might demonstrate using mathematical typicality *strategically*. First, students might realize that conjectures that hold for mathematically atypical objects (i.e., objects with mathematically-special properties) may not hold for all objects. For instance, figuring out that a conjecture holds for the number “1” may not be strong evidence that the conjecture would hold for all numbers, since 1 has very special properties (e.g., being the multiplicative identity). However, this conception of mathematical typicality might be directly at odds with students’ everyday notions of typicality, because although 1 is highly atypical in a mathematical context, it is highly typical in students’ everyday life. Thus if typicality is used strategically, we might expect to see a reversal – we would want students to recognize that a number like 1 is highly *atypical* in a mathematical context, despite

the fact that it is highly *typical* in an everyday context.

Second, students might use mathematical typicality *strategically* if they realize that superficial or “surface-level” features of a mathematical object are not particularly important when considering whether conjectures that hold for that object will hold for most objects. For instance, students might realize that when a parallelogram is in a non-standard orientation, this is unlikely to impact most mathematical conjectures that are characteristic of middle school mathematics. Similarly, a student might realize that the relative size of a number (e.g., whether it’s 3 or 103) or the cultural significance of a number (e.g., the number 13) might not be particularly important. This strategic use of mathematical typicality may again be at odds with everyday notions of typicality – in day-to-day life, students are accustomed to seeing shapes in standard orientation and working with relatively small numbers, so objects that do not conform to these experiences might be considered atypical. Thus we argue that strategic use of typicality requires students to flexibly switch between their “everyday” and “mathematical” lenses.

### Methods

A total of 475 middle school students (46% female) from a suburban middle school in a Midwestern state were included in the study. Students were distributed across grades 6 (144 students), 7 (160 students), and 8 (163 students), and mathematics classes used reform texts. The school demographics were 48% Caucasian, 21% African American, 14% Asian, 11% Hispanic, and 1% Native American, with 37% low income, and 10% English Language Learners (ELL).

A survey was administered to all participants during their normal math classes. Each survey contained questions relating to two of four different domains: numbers, parallelograms, triangles, and birds (birds are omitted from this analysis). For each domain students received, they were presented with specific mathematical objects or items in that domain (e.g., a small equilateral triangle or the number “6”) and asked to rate each item’s typicality on a 1-7 scale. Students rated the typicality of each item in a mathematical context and in an everyday context (Figure 1). The order of the 9 items within each context and the order of the 2 domains were randomized.

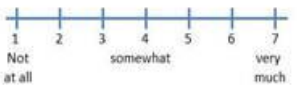
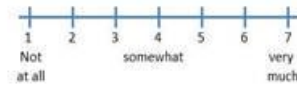
<p><b>Mathematical Context</b>  <i>“Now we want you to think about mathematical properties - that is, the kinds of things you learn about in math class.”</i></p>	<p>Imagine that we learned a new mathematical property that was true of this parallelogram. How likely is it that the property will be true of most other parallelograms?</p>	
<p><b>Everyday Context</b>  <i>“Now we want you to think about parallelogram shapes you see in everyday life outside of school, for example, in objects around your house or outside.”</i></p>	<p>How typical is this parallelogram of those you see in your everyday life?</p>	

Figure 1: Example of questions on middle school survey

Table 1: Mathematical (italics) and everyday (underline) properties entered into model

Numbers	Parallelograms	Triangles
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<i>Prime or perfect square</i> <i>Power of 2 or 10</i> <i>Multiple of 5 or 10</i> <i>Identity properties (i.e., 0 or 1)</i> <i>Relative magnitude (small or large)</i>	<i>Square, rectangle, or rhombus</i> <i>Size (small, large, “skinny”)</i> <i>Orientation (standard, non-standard, left-leaning)</i>	<i>Isosceles, equilateral, scalene</i> <i>Obtuse, acute, right</i> <i>Size (small, large, “skinny”)</i> <i>Orientation (standard, non-standard)</i>
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The data were analyzed using hierarchical linear regression models (Snijders & Bosker, 1999) where repeated observations were nested within students nested within teachers. Three different models, one per domain, were fit to the data. Random effects included student, teacher, and which mathematical object (i.e., which specific number or shape) the question referenced. Fixed effects included context (Figure 1), the mathematical and everyday properties of the object (Table 1), and the interaction of these two terms. Properties that did not have significant main effects or interactions with context were removed. Fixed effects for gender and grade were not significant in any of the models. Mathematical and everyday properties of numbers, parallelograms, and triangles entered into the model are noted in Table 1. Properties were chosen based on the mathematical knowledge of a team of mathematicians, psychologists, and mathematics educators and former K-12 teachers, as well as based on previous results from our studies of inductive reasoning (Knuth et al., 2011; Cooper et al., 2011).

## Results and Discussion

### Number

As can be seen from Table 2, across mathematical and everyday contexts, students rated small numbers as being more typical ( $p < .001$ ) and numbers ending in 5 and powers of 10 as also being more typical ( $p = .020$  and  $p < .001$ , respectively). This suggests two ways in which students might *not* be considering mathematical typicality strategically. First, students seemed to believe that conjectures that hold for mathematically-special numbers, like powers of 10 or multiples of 5, would be *more* likely to hold for other numbers. While these two properties are related specifically to the base-10 system, rather than being intrinsic properties of number like prime-ness, from a mathematical standpoint it still makes sense that properties that hold for these numbers may be *less* likely to hold for other numbers. Second, students rated that conjectures that held for small numbers were *more* likely to hold for other numbers ( $p < .001$ ). Here, the students may have been considering a superficial or mathematically-irrelevant feature as important when considering mathematical conjectures. In both cases, students’ everyday notions of typicality, their familiarity encountering small numbers, multiples of 5, and powers of 10 in their everyday lives, may have influenced their mathematical notions of typicality – whether it makes sense for properties that hold for these numbers to hold for most other numbers. We also see no evidence of the reversal for mathematical typicality that might evidence strategic thinking. Students did not indicate that numbers with special properties – like prime numbers or perfect squares – were atypical in a mathematical context.

**Table 2: HLM analysis of students’ typicality ratings for number**

	Estimate	S.E.	<i>t</i>	<i>p</i>	Sig.
(Intercept)	3.61	0.46	7.70	< .001	***
Mathematical Context	(ref.)				
Everyday Context	0.19	0.17	1.10	0.274	
Large	(ref.)				
Small	0.77	0.17	4.42	< .001	***
Ends with 5	0.68	0.27	2.55	0.020	*

Power of 10	1.32	0.25	5.39	< .001	***
Prime	-0.17	0.22	-0.76	0.445	
Identity (0 or 1)	-0.37	0.35	-1.05	0.298	***
Everyday Context: Small	0.59	0.09	6.67	< .001	***
Everyday Context: Prime	0.33	0.11	3.07	0.001	**
Everyday Context: Identity	0.55	0.16	3.44	< .001	***

\*  $p < .05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

However, looking at the interaction terms in Table 2, we do see evidence that students are at times using mathematical typicality strategically. First, although students rated small numbers as typical regardless of the context, being small had a larger impact on typicality in an everyday context ( $p < .001$ ). This suggests that students may realize that superficial characteristics, like relative size, are less important when considering a number mathematically. Second, students found both prime and the identity numbers more typical in an everyday context ( $p = .001$  and  $p < .001$ ). Thus although students expressed their familiarity with these numbers by giving them high everyday typicality ratings, this familiarity did not inflate mathematical typicality ratings.

### Parallelograms

Across both mathematical and everyday contexts, students rated squares as being more typical (Table 3;  $p = .015$ ). This again suggests that students might not be considering mathematical typicality strategically – students are expressing the belief that properties that hold for squares are more likely to hold for all parallelograms. Here, students’ everyday familiarity with squares might be interfering with viewing a square as a mathematical object that has special mathematical properties (e.g.,  $90^\circ$  angles, congruent diagonals). We also again do not see evidence of the desired reversal – students do not rate mathematically-special parallelograms (like squares or rhombi) as being less typical in a mathematical context.

**Table 3: HLM analysis of students’ typicality ratings for parallelograms**

	Estimate	S.E.	<i>t</i>	<i>p</i>	Sig.
(Intercept)	2.88	0.61	4.7	< .001	***
Mathematical Context	(ref.)				
Everyday Context	0.39	0.27	1.41	0.165	
Standard Orientation	0.43	0.24	1.78	0.080	
Large	(ref.)				
Small	0.19	0.55	0.34	0.710	
Leans Left	-0.51	0.34	-1.52	0.134	
Square	0.88	0.34	2.58	0.015	*
Rectangle	0.63	0.32	1.99	0.055	
Rhombus	0.57	0.40	1.44	0.154	
Everyday Context: Standard Orientation	0.49	0.11	4.49	< .001	***
Everyday Context: Small	-1.04	0.25	-4.20	< .001	***
Everyday Context: Leans Left	0.52	0.15	3.48	< .001	***
Everyday Context: Square	0.55	0.15	3.64	< .001	***
Everyday Context: Rectangle	1.31	0.14	9.30	< .001	***
Everyday Context: Rhombus	0.56	0.18	3.14	0.001	**

\*  $p < .05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

However, looking at the interaction terms, we see considerable evidence that students can use mathematical typicality strategically. Although students rated squares as being typical regardless of the context, squares were considered even more typical in an everyday context ( $p < .001$ ). Similarly, students rated rectangles and rhombi as more typical in an everyday context ( $p < .001$  and  $p = .001$ ). Students seemed to recognize that although these shapes were common in their

everyday lives, this consideration should not inflate their ratings when determining whether properties that hold for these shapes will hold for other shapes. Students also allowed superficial properties of parallelograms – like size, leaning to the left, and orientation – to influence their everyday typicality ratings ( $p < .001$ ), but not their mathematical typicality ratings.

### Triangles

Across mathematical and everyday contexts, students found equilateral, isosceles, and standard orientation triangles more typical (Table 4;  $p = .037$ ,  $p = .004$ ,  $p < .001$ , respectively) and found skinny triangles less typical ( $p = .002$ ). This suggests that students might not be using mathematical typicality strategically. Students expressed that conjectures that hold for special triangles like isosceles and equilateral triangles are more likely to hold in general, and that conjectures that hold for skinny-looking or non-standard orientation triangles, which are superficial considerations, are less likely to hold in general. Here, again, students do not seem to be differentiating between an everyday notion of typicality (the commonness of equilateral and isosceles triangles in their everyday life, and the relative rarity of skinny and non-standard orientation triangles) and a mathematical notion of typicality (whether conjectures that hold for certain triangles are likely to hold for other triangles). We also again see no evidence of the desired reversal – students did not rate mathematically-special triangles as atypical, and thus did not show recognition that conjectures that hold for these triangles are less likely to hold for other triangles.

**Table 4: HLM analysis of students' typicality ratings for triangles**

	Estimate	S.E.	<i>t</i>	<i>p</i>	Sig.
(Intercept)	4.98	0.41	12.16	< .001	***
Mathematical Context	(ref.)				
Everyday Context	0.15	0.20	0.75	0.457	
Skinny	-0.74	0.17	-4.28	0.002	**
Isosceles	0.63	0.17	3.64	0.004	**
Equilateral	0.75	0.33	2.28	0.037	*
Acute	(ref.)				
Obtuse	-0.049	0.19	-0.25	0.794	
Right	0.20	0.27	0.75	0.453	
Standard Orientation	0.59	0.18	3.19	0.009	**
Everyday Context: Equilateral	0.68	0.22	3.19	0.002	**
Everyday Context: Obtuse	-0.16	0.10	-1.56	0.124	
Everyday Context: Right	0.46	0.16	2.91	0.004	**

\*  $p < .05$ , \*\*  $p < .01$ , \*\*\*  $p < .001$

Looking at the interaction terms in Table 4, we see instances in which students seem to be strategically reasoning about mathematical typicality. Although students rated equilateral triangles as typical regardless of context, these triangles were rated as being even more typical in an everyday context ( $p = .002$ ). Further, students rated right triangles as being typical in an everyday context ( $p = .004$ ), but did not let their everyday familiarity inflate their ratings in a mathematical context. This suggests that students may realize that although equilateral and right triangles may be highly salient in their everyday experiences, this everyday familiarity should not affect whether conjectures that hold for these triangles will hold for other triangles.

### Summary and Conclusions

Here we examined whether middle school students use typicality *strategically* when considering conjectures, and we found mixed results. When objects like numbers or shapes had

special mathematical properties, students considered them *more* typical in a mathematical context. However, considering typicality strategically, properties that hold for these objects should be *less* likely to hold for other objects. In other cases, superficial characteristics of objects impacted whether students thought that conjectures that held for that object would hold for other objects. Both behaviors suggest that students might be conflating everyday typicality with mathematical typicality. Despite these results, students did sometimes distinguish between mathematical and everyday contexts; they appropriately recognized the relevance of mathematically-special and surface-level properties within each domain. This suggests that students already have important resources for using typicality strategically, and for differentiating how objects should be considered in the math classroom and everyday life. But are these behaviors really characteristic of mathematical expertise? To get a sense of how mathematics experts would rate the typicality of these items, we presented the survey to 339 mathematicians. Initial analyses suggest that mathematicians do use typicality strategically in the ways we predicted, and they recognize everyday and mathematical typicality as two distinct entities that are often in direct opposition. This stands in contrast to how middle school students considered typicality, as they had difficulty reconciling mathematical and everyday contexts.

Our results suggest that students must negotiate an important learning continuum regarding mathematical conjectures. Initially, students appear to have difficulty reconciling their mathematical experiences with numbers and shapes with their concrete, salient everyday experiences. However, expertise in mathematics is characterized by flexible application of formal mathematical knowledge and everyday experience, based on the features of the problem and the social context. Thus students should be encouraged to critically reflect on how mathematical objects like numbers and shapes are considered differently in the mathematics classroom when exploring conjectures, compared to interacting with these objects in day-to-day life. Our work suggests that mathematicians are able to move flexibly between each of these two viewpoints, and use both examples and typicality judgments as resources in their work. Strategic use of examples and considerations of typicality may thus be important in helping students think more critically about the nature of mathematical evidence and in moving students towards making important generalizations about why mathematical conjectures hold, both of which ultimately could support deductive reasoning and formal mathematical proof.

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### References

- Alcock, L. J. (2004). Uses of example objects in proving. In M. J. Hines & A. B. Fuglestad (Eds.), *Proceedings of the 28<sup>th</sup> annual conference of the International Group for the Psychology of Mathematics Education (Vol. 2)*. Bergen, Norway: Bergen University College.
- Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, 24(4), 359- 387.
- Common Core State Standards Initiative. (2010). *Common Core State Standards (Mathematics Standards)*.
- Cooper, J., Walkington, C., Williams, C., Akinsiku, O., Kalish, C., Ellis, A., & Knuth, E. (2011). Adolescent reasoning in mathematics: Exploring middle school students' strategic approaches to empirical-based justifications. In *Proceedings of the 33rd Annual Conference of the Cognitive Science Society*. Boston, MA.



- Feeny, A., & Heit, E. (2007). *Inductive reasoning: Experimental, developmental, and computational approaches*. Cambridge: Cambridge University Press.
- Greeno, J. (2006). Learning in activity. In R. K. Sawyer (ed.) *The Cambridge Handbook of the Learning Sciences* (pp.79-96). St. Louis, Cambridge University Press.
- Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *Issues in Mathematics Education*, 7, 234-283.
- Healy, L., & Hoyles, C. (2000). A study of proof conceptions in algebra. *Journal for Research in Mathematics Education*, 31(4), 396-428.
- Knuth, E., Kalish, C., Ellis, A., Williams, C., & Felton, M. (2011). Adolescent reasoning in mathematical and non-mathematical domains: Exploring the paradox. In V. Reyna, S. Chapman, M. Dougherty, & J. Confrey (Eds.), *The adolescent brain: Learning, reasoning, and decision making*. Washington, DC: American Psychological Association.
- Masingila, J., Davidenko, S., & Prus-Wisniewska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics*, 31, 175-200.
- Moll, L., & Gonzalez, N. (1997). Teachers as social scientists: Learning about culture from household research. In P. M. Hall (Ed.), *Race, ethnicity, and multiculturalism: Policy and practice* (pp. 89-114). New York: Garland.
- Moses, R., & Cobb, C. (2001). *Radical Equations: Math Literacy and Civil Rights*. Boston: Beacon Press.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA
- Osherson, D.N., Smith, E.E., Wilkie, O., Lopez, A., & Shafir, E. (1990). Category-based induction. *Psychological Review*, 97, 185-200.
- Reusser, K., & Stebler, R. (1997). Every word problem has a solution: The social rationality of mathematical modeling in schools. *Learning and Instruction*, 7(4), 309-327.
- Snijders, T. & Bosker, R. (1999). *Multilevel Analysis: An Introduction to Basic and Advanced Multilevel Modeling*. Sage Publications.
- Walkington, C., Nathan, M., Wolfgram, M., Alibali, M., & Srisurichan, R. (in press). Bridges and barriers to constructing conceptual cohesion across modalities and temporalities: Challenges of STEM integration in the precollege engineering classroom. Chapter in *Engineering in PreCollege Settings: Research into Practice*. Johannes Strobel, Senay Purzer, Monica Cardella, (Eds.), Sense Publishers.
- Walkington, C., & Sherman, M. (2012). Using adaptive learning technologies to personalize instruction: The impact of interest-based scenarios on performance in algebra. To appear in *Proceedings of the 10th International Conference of the Learning Sciences*. Sydney, Australia.
- Walkington, C., Sherman, M., & Petrosino, A. (in press). 'Playing the game' of story problems: Coordinating situation-based reasoning with algebraic representation. *Journal of Mathematical Behavior*.
- Williams, C., Akinsiku, O., Walkington, C., Cooper, J., Ellis, A., Kalish, C., Knuth, E. (2011). Understanding students' similarity and typicality judgments in and out of mathematics. In *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Reno, NV.