

# The Associations Between Readability Measures and Problem Solving in Algebra

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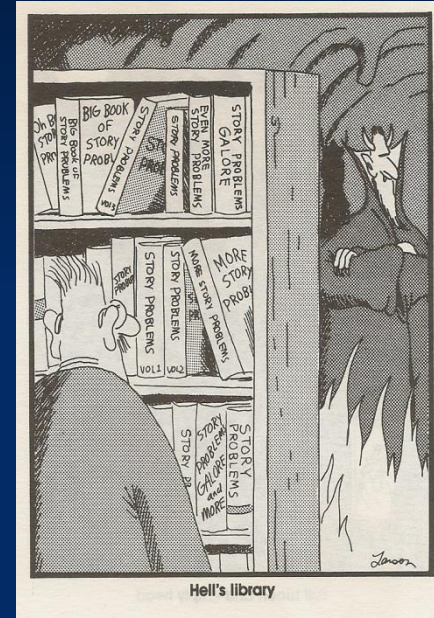
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# Introduction

- Mathematics story problems are sometimes considered notoriously difficult (Cummins et al., 1988)
- However, even simple/traditional story problems can play an important role in mathematics learning (Walkington, Sherman, & Petrosino, 2012):
  - Provide **access** to mathematical ideas through **grounding** (Goldstone & Son, 2005) in verbal or situational knowledge
  - Enhance motivation for or **interest** (Hidi & Renninger, 2006) in engaging with mathematics



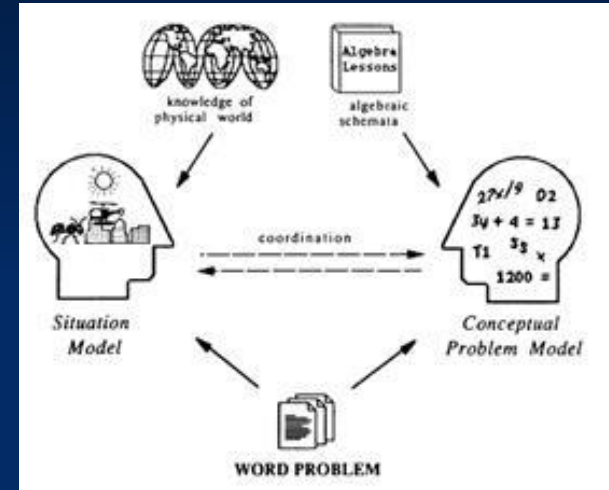
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# Comprehending Story Texts

- In order to understand and solve story problems, learners must comprehend the problem text (Kintsch & Greeno, 1985)
- Successful problem-solvers coordinate **situation model** (understanding of actions and relationships) with **problem model** (formal equations) (Walkington, Sherman, & Petrosino, 2012)
- Both the **readability level** and the **topics** of story problems may impact success, in addition to mathematical skills



Nathan, Kintsch, &  
Young (1992)



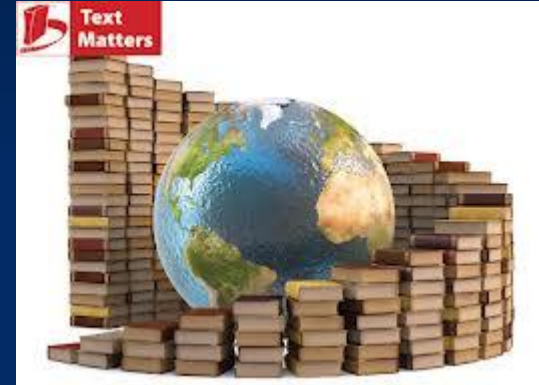
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# Readability of Story Problems

- Research has shown that reading skill is a strong predictor of performance on math story problems (Kyattla & Bjorn, 2013)
- Research has not successfully linked traditional readability measures (words, syllables, etc.) to mathematics performance (Wiest, 2003)
- Lots of research linking use of mathematical/relational language in problems to problem-solving success (Clement, 1982; Cummins et al., 1998; Hegarty et al., 1995; Lewis & Mayer, 1987; Pape, 2004; Vincente, Orrantia, & Verschaffel, 2007)



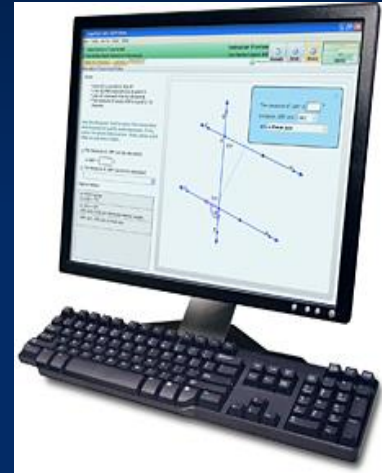
# Topics of Story Problems

- Some problem topics may be more or less familiar to students
  - Urban students in the South solving problems about shoveling snow or building a greenhouse (Walkington, 2010)
- Matching problem topics to students' interests and experiences (i.e., **context personalization**) can improve both immediate performance and long term learning in mathematics (Walkington, 2013)



# Research Questions

- **RQ1**: How are quantitative measures of **text readability** associated with problem-solving performance on algebra story problems?
- **RQ2**: How are quantitative measures of **topic incidence** associated with problem-solving performance on math story problems?



# Participants

- 9 high schools and 1 middle school using Cognitive Tutor Algebra (CTA) curriculum ( $N = 3394$  students)

School Characteristic	Schools in Sample
Geographic Location	10 different states, 2 rural, 4 urban, and 2 suburban
F/R Lunch %	3 had 0-33% of students eligible, 4 had 33-66% eligible, 2 had 66-100% eligible
Race/Ethnicity	5 predominately White , 3 predominantly African-American, 2 predominately Hispanic
Achievement Level	3 had under 50% of students proficient, and 3 had 80% or more proficient, 4 in between

# Cognitive Tutor Algebra

The screenshot shows the Cognitive Tutor Algebra interface. The top menu bar includes 'File', 'Tutor', 'Go To', 'View', and 'Help'. The main title bar reads '8 - Linear Models and Independent Variables' and '1 - Finding Independent Variables with Positive Rates of Change'. The 'Instructor Preview' section shows 'BH102' and buttons for 'Example', 'Hint', 'Done', and 'Skills'. The 'Scenario' pane on the left contains the following text:

You have just been promoted to assistant manager at PAT-E-OH Furniture Inc. and have received a raise to \$10.50 per hour.

- How much would you be paid if you worked five hours?
- How much would you be paid if you worked 10 and 1/2 hours? If you have not already done so, please fill in the expression row with an algebraic expression for the total pay. Then use the expression and the Solver to answer questions 3 and 4 below.
- How many hours must you work to make five hundred fifty dollars?
- In order to make \$2,200.00, how many hours must you work?

To write the expression, define a variable for the time worked and use this variable to write a rule for your total pay.

The 'Worksheet' pane on the right contains a table for student input:

Quantity Name		
Unit		
Expression		
Question 1		
Question 2		
Question 3		
Question 4		

Below the input table is the 'Answer Key:'

Quantity Name	the time worked	the money earned
Unit	hour	dollar
Expression	X	10.5X
Question 1	5	52.5
Question 2	10.5	110.25
Question 3	52.381	550
Question 4	209.5238	2200

Data collected from the first 8 units in CTA on linear functions – 151 unique story problems



# Readability: Coh-Matrix

- Provides 108 quantitative measures of (McNamara et al., in-press):
  - **Surface code**: Difficulty of words and syntax
  - **Textbase**: Ease of connecting different ideas in the text
  - **Situation model**: Consistency of various dimensions of the mental representation of the text such as causation, time, and space

## Coh-Matrix

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*Coh-Matrix calculates the coherence of texts on a wide range of measures. It replaces common readability formulas by applying the latest in computational linguistics and linking this to the latest research in psycholinguistics.*

# Topic Incidence - LiWC

- Dictionary-based text analysis program (Pennebaker, Chung, Ireland, Gonzales, & Booth, 2007)
- The topic of a story problem was determined by whether it had at least one word that fell in one of the LIWC topics or sub-topics

LIWC Topics	LIWC Sub-Topics
<b>Social</b>	family, friends, people
<b>Affective</b>	positive emotions and negative emotions
<b>Biological</b>	body, health, ingestion
<b>Cognitive</b>	insight, causation, discrepancy, tentativeness, certainty, inhibition, in/exclusiveness
<b>Perceptual</b>	see, hear, feel
<b>Relativity</b>	motion, space, time
<b>Personal</b>	work, achievement, leisure, home, money, religion, death

# Analysis Techniques

- Logistic regression models predicting: **Correct, Hint, Incorrect**
- Linear regression model predicting: **Latency**
- Tested only readability/topic indicators with significant correlations to one or more of outcomes
- Sample size was  **$N = 151$  problems** - each problem's performance measures were averaged for all students who solved that problem
- Random effects to control for mathematical aspects of problem, as well as for student effects
- Predictors tested for inclusion using Chi-Square test for significant reductions in deviance of the model

# Results - Readability

- **Moving from 3 to 4 sentences** reduced corrects by an estimated 4.4% ( $p < .01$ ), increased incorrects by an estimated 2.75% ( $p < .05$ ) and increased hints by an estimated 1.4% ( $p < .01$ )
- **Third person singular pronouns** (3PS; he/she/it) associated with more correct answers ( $p < .05$ ) and fewer hints ( $p < .01$ )
  - Changing a problem with no 3PS to a problem that has 10% of its words as 3PS should increase correct answers by an estimated 3.6%
- Using **words with multiple meanings** (word polysemy) significantly increased latency ( $p < .05$ )

# Results - Readability

Category	High Example	Low Example
Third person singular pronouns	A training sumo wrestler Tu Fatmo weighs 470 pounds. <u>He</u> is 80 pounds below his ideal fighting weight. <u>He</u> can safely gain four and one-half pounds per week. (incidence = 107.143 per 1000 words)	In the Coast-to-Coast Bicycle Classic people cycle 3500 miles from Seattle Washington to New Jersey. Their target is to ride an average of 80 miles each day. The riders think a lot about how far they have left to go and whether they can make the trip all the way across the country. (incidence = 0 per 1000 words)
Word polysemy	An open <u>pit</u> copper <u>mine</u> is 1550 feet deep and the company estimates that it is getting deeper at the rate of seven <u>feet</u> per month. Assume the number of feet below the surface is a negative number. (Average number of meanings of each content word = 6.375)	On Tuesday morning at 7 AM the residents of Bar Harbor Maine awoke to six inches of snow on the ground. The snow fell at the average rate of one-half inch per hour during the storm. (Average number of meanings of each content word = 2.342)

# Results – Topic Measures

- Words involving **social processes** decrease incorrect answers ( $p < .05$ ) by an estimated 2.09%
  - References to family, friends, humans, socializing, speaking, having parties, making calls, sending messages
- **Work words** decrease corrects ( $p < .05$ ) by an estimated 1.89%
- **Motion words** decrease corrects ( $p < .05$ ) by an estimated 2.03%
- **Healthcare words** increase hints ( $p < .01$ ) by an estimated 1.63%
- Inhibition words (**saving money**) increase hints ( $p < .05$ ) by an estimated 0.98%

# Results – Topic Measures

Topic	Example Problem
<b>Social</b>	A bride is making nameplates to put on the tables at her reception. She can make them at the rate of 25 per hour. She works for two hours and quits for the night realizing that she cannot complete this many nameplates herself. The next day she calls her mother and they both work together. Her mother can make 35 nameplates per hour.
<b>Work</b>	You have just been promoted to assistant manager at PAT-E-OH Furniture Inc. and have received a raise to \$10.50 per hour.
<b>Motion</b>	A machine called the Crawler which moves space shuttles travels at the rate of 29 feet per second. The Crawler is currently 100 feet from the hanger moving toward the launching pad.
<b>Health</b>	According to the American Heart Association approximately 145000 women die every year from smoking-related diseases. In fact lung cancer has become the leading cause of cancer death among women.
<b>Inhibition</b>	During the school year teachers save money for use during the summer when they're not being paid. This year due to some unexpected expenses one teacher was able to save only \$879. He figures he will need \$23 a day for personal spending money.

# Discussion

- **Good** for problem solving performance: Shorter stories, third person singular pronouns, social contexts
- **Bad** for problem solving performance: Longer stories, words with multiple meanings, healthcare, business, financial, or physics contexts
- **Effect sizes are small** – readability or topic predictors only reduce model deviance by around 3% when predicting correct answers
- Makes sense that ***student*** characteristics and ***mathematical*** characteristics of the problem should explain most of performance



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# Implications & Future Directions

- In algebra, need to learn how to navigate complex and potentially lengthy story contexts on diverse array of real world/professional topics
- To introduce and provide access to **new, challenging ideas**, readability and topic findings can be taken into account by problem designers
- Fairness of large scale assessments measuring math learning
- **Future Directions** – look for larger effects by:
  - Examining results by problem step (label quantities, write equation, etc.)
  - Examining results for students in 3 low-performing schools
  - AERA Research Grant – larger bank of more diverse problems



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Questions? Comments?



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