

GIVING SYMBOLS SITUATION- BASED MEANING: CONNECTING REPRESENTATIONS IN ALGEBRA I

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CCSS (2010) – ALGEBRA I

In Algebra I, students must learn to work with **symbolic representations**:

- ▶ *“Interpret the structure of expressions.”*
- ▶ *“Create equations that describe numbers or relationships.”*
- ▶ *“Solve equations in one variable.”*
- ▶ *“Understand solving equations as a process of reasoning and explain the reasoning.”*
- ▶ *“Interpret expressions that represent a quantity in terms of its context.”*

SYMBOLIZATION IN ALGEBRA

The distance in feet that a machine called the Crawler has traveled from its hangar is given by the equation $y = 4x + 175$, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler be a total of 275 feet from the hangar?

The image shows handwritten algebraic work on a white background. At the top, the equation $275 = 4x + 175$ is written. Below it, the number 175 is subtracted from both sides, indicated by a horizontal line and a minus sign. This results in $100 = 4x$. Below this, the number 4 is written under 100, and a horizontal line is drawn. The result is $25 = x$.

What **probing questions** might you ask this student, to assess if they understand key concepts related to using symbolic expressions?

Walkington, Sherman, &
Petrosino (in press)

SYMBOLIZATION IN ALGEBRA

The distance in feet that a machine called the Crawler has traveled from its hangar is given by the equation $y = 4x + 175$, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler be a total of 275 feet from the hangar?

$$\begin{array}{r|l} 275 = 4x + 175 & \\ + 175 & - 175 \\ \hline \frac{100}{4} & = \frac{4x}{4} \\ & \\ & 25 = x \end{array}$$

What could the “4” represent in the given situation? What could the “175” represent?

SYMBOLIZATION IN ALGEBRA

The distance in feet that a machine called the Crawler has traveled from its hangar is given by the equation $y = 4x + 175$, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler be a total of 275 feet from the hangar?

Handwritten algebraic solution for the equation $275 = 4x + 175$. The work shows the equation with 175 subtracted from both sides, resulting in $100 = 4x$. This is then divided by 4 to find $25 = x$.

$$\begin{array}{r|l} 275 = 4x + 175 & \\ + 175 & - 175 \\ \hline \frac{100}{4} & = \frac{4x}{4} \\ & \\ & 25 = x \end{array}$$

“It could have already started at 4 feet... and 175 could equal the time it took for it to go 4 feet.”

Walkington, Sherman, &
Petrosino (in press)

SYMBOLIZATION IN ALGEBRA

*“In general, if students engage extensively in symbolic manipulation before they develop a **solid conceptual foundation** for their work, they will be unable to do more than mechanical manipulations.”*
(NCTM, 2000)

CONCERT PROBLEM (10 MIN)

You are working for a local concert hall. You've determined that the profit the concert hall makes is a function of the price, x , of a ticket, where the profit function is given by:

$$P(x) = -75x^2 + 1500x - 4800$$

Your boss wants you to explain to her exactly what the equation means, so you rewrite the profit function as:

$$P(x) = x(1500 - 75x) - 4800$$

CONCERT PROBLEM

$$P(x) = x(1500 - 75x) - 4800$$

Profit

Price per
ticket

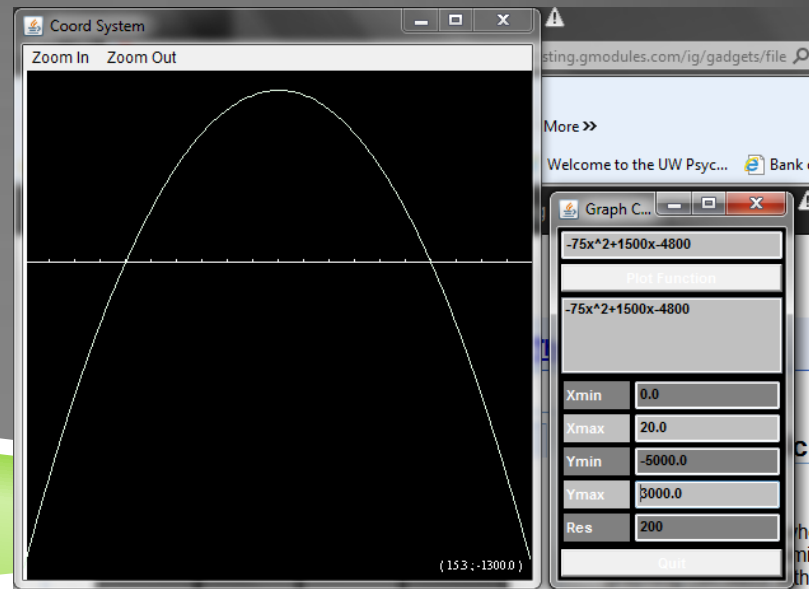
Number of
tickets sold

Fixed costs

SOLUTION STRATEGIES

- ▶ Multiple symbolic representations
- ▶ Construct table using spreadsheet tool
- ▶ Construct graph using graphing calculator ([here](#))
- ▶ Unit analysis

1 Concert Problem				
2	Price per ticket	Number of tickets sold	Revenue	Profit
3	0	1500	0	-4800
4	1	1425	1425	-3375
5	2	1350	2700	-2100
6	3	1275	3825	-975
7	4	1200	4800	0
8	5	1125	5625	825
9	6	1050	6300	1500
10	7	975	6825	2025
11	8	900	7200	2400
12	9	825	7425	2625
13	10	750	7500	2700
14	11	675	7425	2625
15	12	600	7200	2400
16	13	525	6825	2025
17	14	450	6300	1500
18	15	375	5625	825
19	16	300	4800	0
20	17	225	3825	-975
21	18	150	2700	-2100
22	19	75	1425	-3375
23	20	0	0	-4800
24				



CONCERT PROBLEM

- ▶ How do you think the activity could support students learning to use symbolic representations with understanding?
- ▶ How were symbols given meaning?

GIVING SYMBOLS MEANING

▶ **The Concert Problem...**

- ▶ Encourages students to think about what the symbols and parameters mean in terms of a real situation (*Goldstone & Son, 2005*)
- ▶ Allows students to reason structurally about a function, rather than simply dive into well-rehearsed calculations (*Sfard & Linchevski, 1994*)
- ▶ Fosters critical connections between representational formats (*Brenner et al., 1997; Chazan, 1999*)

CONCERT PROBLEM

Your boss decides that she wants to see the profit function with **number of tickets sold** as the independent variable, rather than the **cost per ticket**. Can you fix the function?"

- ▶ Issue that came up with former students

Historic Hotels

Mr. Frank Graham, from Elkhart District in Indiana, has just inherited a historic hotel.

Write a letter to Mr. Graham telling him what price to charge for the rooms to maximize his profit and include your procedure for him to use in the future.



CONCERT PROBLEM

Your boss decides that she wants to see the profit function with **number of tickets sold** as the independent variable, rather than the **cost per ticket**. Can you fix the function?"

- ▶ Issue that came up with former students
- ▶ **Discuss:** Conceptually, how might you approach “fixing” the function?

CONCERT PROBLEM

$$P(x) = x(1500 - 75x) - 4800$$

Profit **Price per ticket** **Number of tickets sold** **Fixed costs**

CONCERT PROBLEM

$$(1500 - 75x)$$

Number of
tickets sold

$$\text{no. tickets sold} = 1500 - 75 * \text{price}$$

$$n = 1500 - 75p$$

$$p = \left(-\frac{1}{75} \right) n + 20$$

CONCERT PROBLEM

- ▶ Reasoning through the **meaning** of symbols and parameters allows for this difficult transformation
- ▶ It also addresses questions like:
 - ▶ How many fewer tickets do you sell for every \$1 you increase the price?
 - ▶ What's the maximum number of tickets you could possibly sell?
- ▶ Supports performing **maximization/break even** calculations with understanding

CCSS (2010) – ALGEBRA I

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CCSS FOR MATHEMATICAL PRACTICE

#2 Reason abstractly and quantitatively

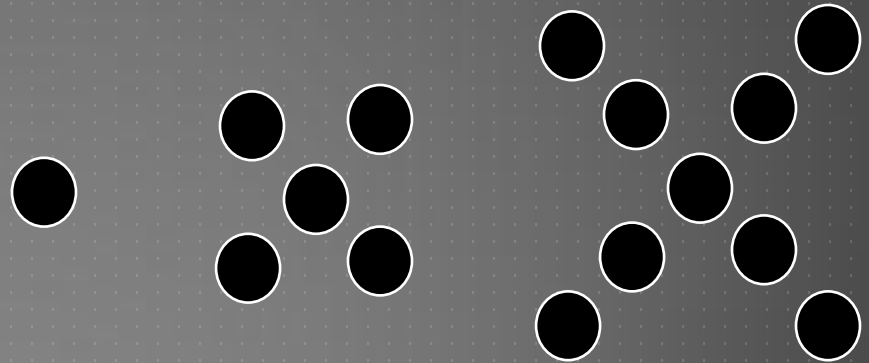
- ▶ Mathematically proficient students make sense of quantities and their relationships in problem situations.
- ▶ They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own - and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

IF TIME ALLOWED...

VIDEO

- ▶ High school mathematics class learning about linear functions
- ▶ Students have been working on problem, now presenting to class

Describe the pattern.
Assuming the sequence continues in the same way, how many dots are there at 100 minutes?



number of dots = $4m + 1$

VIDEO: DISCUSSION POINTS

- ▶ How are symbols tied to their meaning in this activity?
- ▶ How do multiple representations of the key algebraic idea emerge in this class and how are they connected?

(SHOW VIDEO)

VIDEO: DISCUSSION POINTS

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- ▶ How do multiple representations of the key algebraic idea emerge in this class and how are they connected?

VIDEO: DISCUSSION POINTS

- ▶ Connections between multiple representations
- ▶ Diverse ways of defining IVs
- ▶ Give meaning to symbols through explanations
- ▶ Importance of discussion, interpreting students' mathematical work

TICKET OUT

- ▶ What are some of the challenges that might be involved when encouraging students to engage in making sense of symbolic representations?
- ▶ What do YOU think some of the payoffs might be of promoting such meaning-making?

THANK YOU VERY MUCH!